

Suggested Solution to Project 4

Models of the Spread of the Disease Among Rabbits in Meckenburg-West Pomerania

a) The linear model.

$$\frac{dP}{dt} = 30.2,$$

$$P = 30.2t + C$$

$$\text{Use } P(0) = 95 \Rightarrow C = 95$$

$$P(t) = 30.2t + 95.$$

The predicted number of cases of the disease by 12 June, 2003 ($t = 87$) is $P(87) = 2722$.

b) The exponential model

$$\frac{1}{P} \frac{dP}{dt} = 0.12,$$

$$\ln|P| = 0.12t + C$$

$$P(t) = Ae^{0.12t}, \quad A = e^C$$

$$\text{Use } P(0) = 95 \Rightarrow A = 95$$

$$P(t) = 95e^{0.12t}.$$

The predicted number of cases of the disease by 12 June, 2003 ($t = 87$) is $P(87) = 3,249,000$.

c) The logistic model

$$\frac{1}{P} \frac{dP}{dt} = 0.19 - 0.0002P = 0.0002(950 - P)$$

$$\frac{dP}{P(950 - P)} = 0.0002dt$$

$$\frac{1}{P} + \frac{1}{950 - P} = \frac{950}{P(950 - P)}$$

Using partial fraction decomposition on the left-hand side and multiplying both sides by 950, we get

$$\left[\frac{1}{P} + \frac{1}{950 - P} \right] \frac{dP}{dt} = 0.19$$

$$\ln|P| - \ln|950 - P| = 0.19t + C$$

$$\ln\left|\frac{P}{950 - P}\right| = 0.19t + C$$

$$\ln\frac{950 - P}{P} = -0.19t - C$$

$$\frac{950 - P}{P} = e^{-0.19t - C}$$

$$\frac{950}{P} - 1 = Ae^{-0.19t}, \quad A = e^{-C}$$

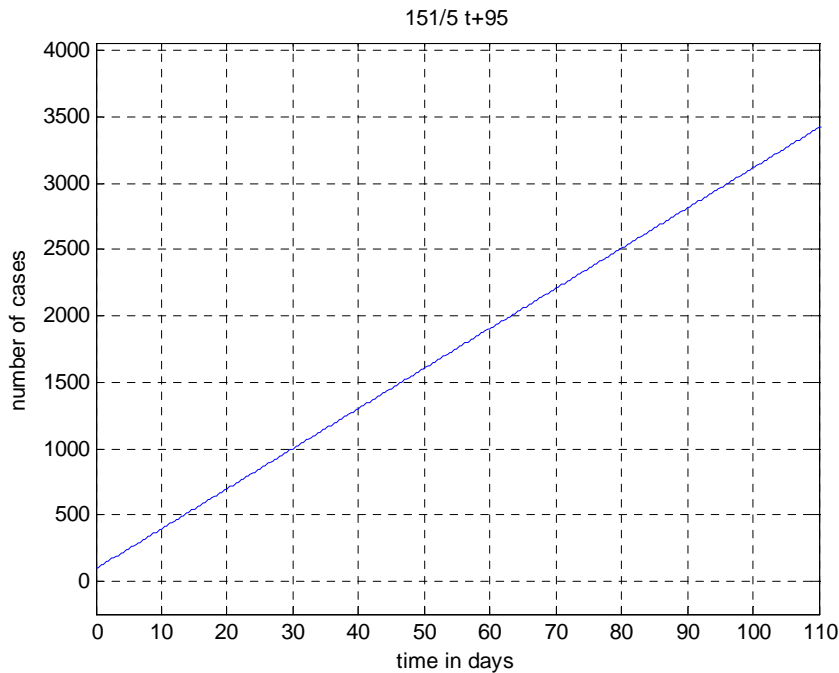
$$P = \frac{950}{1 + Ae^{-0.19t}} \text{ is the general solution.}$$

Using $P(0) = 95$, we find A : $95 = \frac{950}{1 + A} \Rightarrow A = 9$

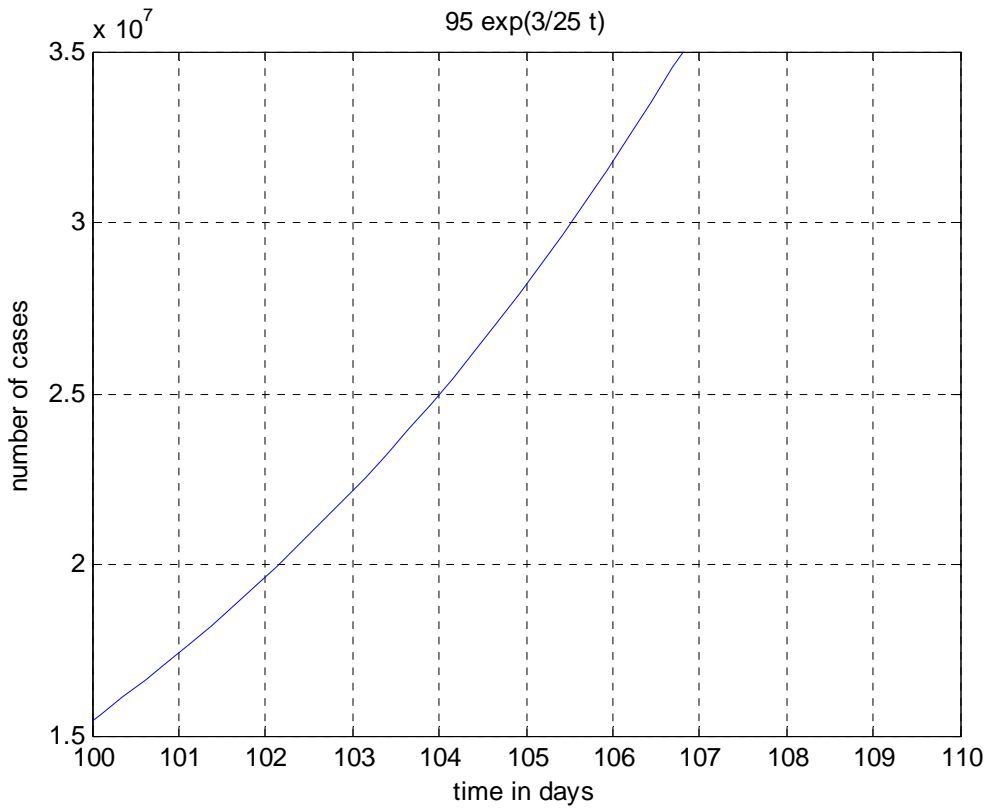
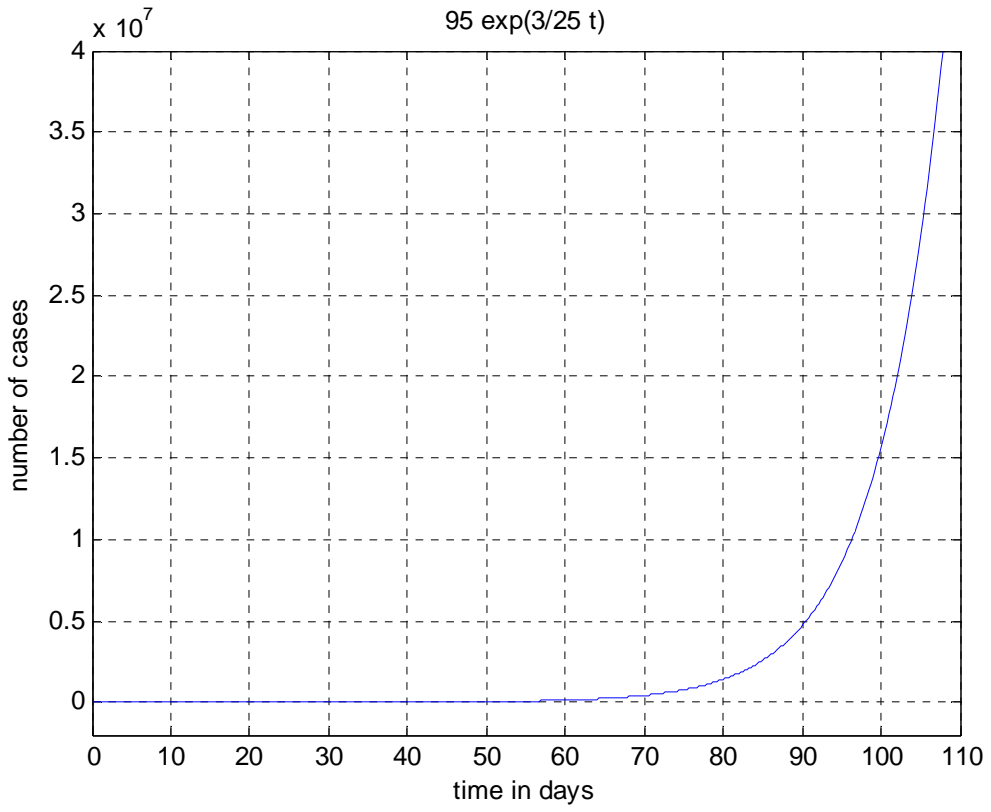
From here $P = \frac{950}{1 + 9e^{-0.19t}}$.

The predicted number of cases of the disease by 12 June, 2003 ($t = 87$) is $P(87) = 950$.

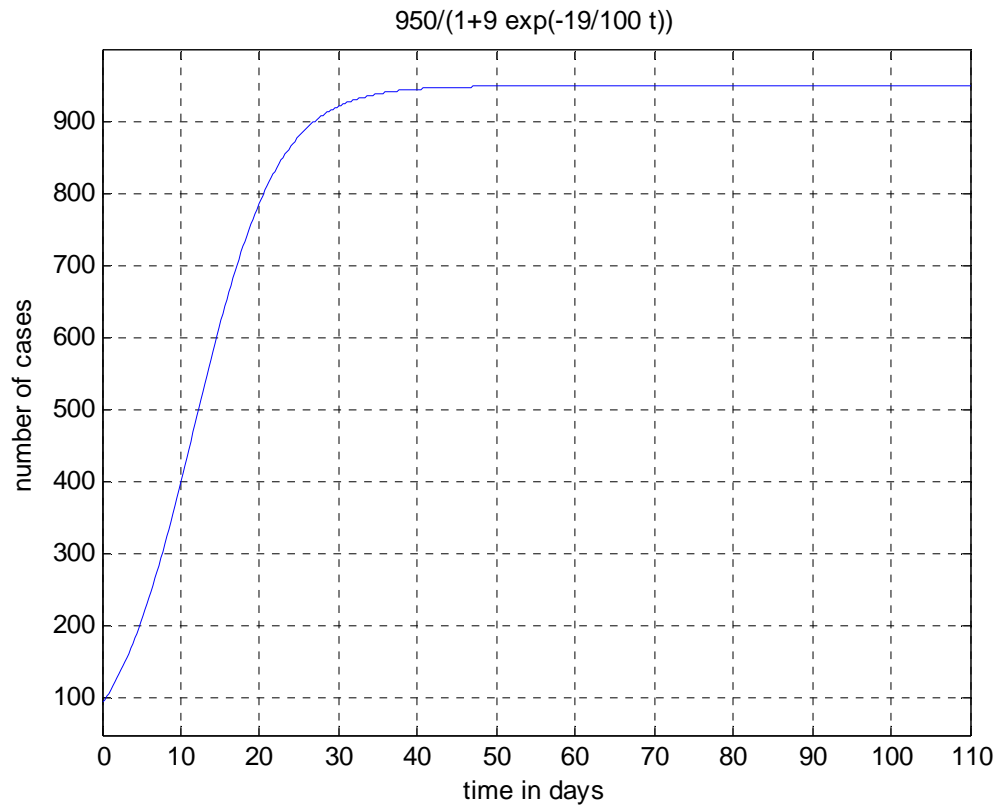
2. a)



b)



c)



3. From the graphs we can read the predicted number of cases of the disease by June 30, 2003 ($t = 105$) for each model:
- a) 3300
 - b) 28 million (see the second graph for the model)
 - c) 950.
4. a) People who created the models might have access to different data (e.g. from newspapers, local government offices, ecological organizations, etc.)
- b) Since the disease was unknown they could take different views on it – pessimistic and optimistic.